Spectral characteristics of UHF radar aurora

B. J. Jackel, D. R. Moorcroft, J. C. Foster, and K. Schlegel

1. Introduction

Coherent backscatter from the auroral ionospheric region, radar aurora, was first detected by Harang and Stoffregen [1938]. Although spectral studies of radar auroral echoes have been made since the 1950s it was only in the late 1960s that technological advances made spectral observations a common part of radar auroral work.

Many studies have been made of radar aurora at VHF (30 – 300 MHz), but in this paper we concentrate on the spectral characteristics of the echoes observed in the UHF band (300 – 3000 MHz). Radar aurora in these two frequency ranges differ in several ways. At VHF the spectra are quite variable, leading to the identification of several echo “types” [Haldoupis, 1989], while UHF echoes can perhaps be better viewed as variations on one basic theme [Moorcroft, 1996b]. Radar auroral observations at UHF are largely free of the complicating effects of refraction, density gradients, and complex spectral behavior (all important considerations in the lower VHF range), although a theoretical understanding of observations at UHF is complicated by the increasing importance of kinetic theory effects.

As is well known, radar aurora has a strong dependence on the magnetic aspect angle, the angle between the radar line of sight and the plane perpendicular to the magnetic field. In this paper we will refer to aspect angles less than $3^\circ$ as small, and those greater than $3^\circ$ as large. Previous UHF spectral studies at small aspect angles include: those using radars at Millstone Hill, both at 440 MHz [St.-Maurice et al., 1989; Foster and Tetenbaum, 1991, 1992; Foster et al., 1992; del Pozo et al., 1993] and at 1295 MHz [ABEL and NEWELL, 1969; HAGFORS, 1972], the former Homer, Alaska radar, operating at 398 MHz [MOORCROFT and TSU NODA, 1978; HALL and MOORCROFT, 1988; MOORCROFT, 1996b], and the CW COSCAT radar at 933 MHz [MCCREA et al., 1991; EGLITIS et al., 1995]. Large aspect angle measurements of spectra have been made using EISCAT (933 MHz) in a number of studies [MOORCROFT and SCHLEGEL, 1988; SCHLEGEL et al., 1990; JACKEL et al., 1997], and using the 440 MHz Millstone Hill radar [Foster et al., 1992].

In this paper we present the results from UHF spectral observations made with the 933 MHz EISCAT (European Incoherent Scatter) radar system, the 933 MHz COSCAT (Coherent Scatter) radar system, and a new 440 MHz bistatic radar system using the Millstone Hill incoherent scatter radar as transmitter, and a mobile receiving system at two different locations in Canada, referred to in this paper as the MIDASC (Millstone Hill Data Acquisition System - Canada) system. Observations from both large and small magnetic aspect angles will be presented, and a new approach to spectral analysis will be used to analyze all
these data, including a reanalysis of some previously reported observations from EISCAT.

[6] We find that nearly all UHF spectra share certain basic characteristics and can be modelled with parameters that vary significantly with aspect angle but not radar frequency. This well ordered behavior is in sharp contrast to the situation at VHF. Interestingly, UHF spectra appear to be similar in many ways to those at HF, despite the completely different plasma scale size involved. Although there is currently no detailed theoretical framework for understanding UHF spectra, our results provide direction and constraints for the future development of such theories.

2. Analysis of Spectral Information

[7] The radar auroral echo strength is proportional to the Fourier component of the electron density with wavelength

\[ \lambda_p = \lambda_f / 2 \cos(\gamma/2) \]  

(1)

where \( \lambda_f \) is the radar wavelength and \( \gamma \) is the scattering angle between the directions from the scattering volume to the transmitter and receiver. For a monostatic radar \( \gamma = 0 \) and \( \lambda_p = \lambda_f / 2 \). It is common to express spectral frequency in terms of the equivalent Doppler velocity \( \nu = -\lambda_p f \), where \( f \) is the Doppler frequency, so that a negative Doppler shift corresponds to positive velocities (motion away from the radar). The opposite sign convention is also frequently used, and care must be taken when comparing results from different studies.

2.1. Spectral Moments

[8] Spectra are often described in terms of their first few moments. These give the total power, \( P_0 \), the mean frequency, \( \bar{\nu} \), the standard deviation, \( \sigma \), and the third moment, which is often expressed as the dimensionless quantity skewness, represented here by the symbol \( S_0 \). In this study it was found convenient to express the third moment in terms of two other quantities closely related to \( S_0 \). The first, \( S \), is obtained by multiplying \( S_0 \) by minus the sign of the third moment. The second, represented by the symbol \( S^* \), is proportional to the cube root of \( S \) and has the same dimensions as \( \bar{\nu} \) and \( \sigma \). If \( P(\omega) \) is the Doppler power spectrum, then

\[ P_0 = \int_{-\infty}^{+\infty} d\omega P(\omega), \]  

(2)

\[ \bar{\nu} = \frac{1}{P_0} \int_{-\infty}^{+\infty} d\omega \omega P(\omega), \]  

(3)

\[ \sigma^2 = \frac{1}{P_0} \int_{-\infty}^{+\infty} d\omega (\omega - \bar{\nu})^2 P(\omega), \]  

(4)

\[ S_0 = \frac{1}{\sigma^3 P_0} \int_{-\infty}^{+\infty} d\omega (\omega - \bar{\nu})^3 P(\omega), \]  

(5)

\[ S = \left( \frac{S^*}{\sigma} \right)^{3/2} \frac{\bar{\nu}}{\bar{\nu}} S_0, \]  

(6)

[9] As can be seen from equations 2–6, the factor \( (\omega - \bar{\nu})^3 \) gives most weight to those parts of the spectrum furthest from the mean where the signal-to-noise ratio (SNR) is often smallest. As a result, noise in those parts of the spectrum may distort or even dominate the estimates of standard deviation and skewness. Most modern radars, including those used in the present study, initially obtain their spectral information as measurements of the autocorrelation function (ACF) which are Fourier transformed to obtain power spectra. The difficulties just mentioned in calculating moments of power spectra can be partly circumvented by estimating moments directly from the ACFs [Jackel, 2000]. Although noise subtraction is normally available for radar data (including that treated in this paper), that is never 100% effective. It can prove easier to deal with residual noise in the lag domain. It is also computationally simpler to estimate spectral moments directly from the ACF. As a practical matter, it is convenient to express \( \bar{\nu} \), \( \sigma \), and \( S^* \) as velocities; we use the notation \( \nu = (\lambda_f/2\pi)\bar{\nu} \), \( \sigma = (\lambda_f/2\pi)\sigma \), and \( S^* = (\lambda_f/2\pi)S^* \).

[10] Regardless of how they are calculated, spectral moments can be clearly interpreted only if the spectra from which they are derived have relatively simple shapes consisting of only a single peak. Multiple peaks in UHF auroral spectra are readily identified by the corresponding anomalously large spectral widths, as well as through the presence of undulations in \( p(\tau) \), the ACF magnitude. In fact, the vast majority of spectra in the data sets used for the present study are single peaked.

2.2. Models of the ACF Magnitude

[11] Jackel [2000] found that experimental measurements of UHF radar auroral ACF magnitudes, \( \rho(\tau) \), were generally well modelled by the two parameter function

\[ \rho(\tau) = \rho(0)e^{-(\tau/\tau_c)^{n_c}}. \]  

(7)

The parameter \( \tau_c \) is a measure of the width of the function \( \rho(\tau) \), while \( n_c \) controls the shape. If \( n_c = 2 \) then the corresponding power spectrum is Gaussian, while \( n_c = 1 \) produces a Lorentzian spectrum. As will be shown, UHF radar auroral ACFs tend to have values of \( n_c \) which fall between these two limits. It is evident that the parameters of this model are not independent of the spectral moments. In particular, for a given value of \( n_c \), there will be an inverse relationship between \( \sigma \) or \( \sigma_0 \), and \( \tau_c \). This will become evident when the observations are presented.

3. Experimental Arrangements

[12] In this section we present some details about the radar systems and configurations experiments that were used, focusing most attention on the Millstone Hill bistatic experiments, since they have not been described previously in the literature.

3.1. Millstone Hill Bistatic Experiments

[13] Although primarily an incoherent scatter radar, for a number of years the 440 MHz Millstone Hill radar has also been used for studies of coherent backscatter from the auroral \( E \) region [St.-Maurice et al., 1989; Foster and Tetenbaum, 1991; 1992; Foster et al., 1992; del Pozo et al., 1993]. It consists of a 2–5 MW transmitter feeding a 46 m diameter steerable antenna. The computer-based receiving system for the radar is known as MIDAS (Millstone...
Hill Data Acquisition System). A fully portable version of this receiving system was constructed and has been operated at two different sites in Canada as a bistatic receiver for the Millstone Hill radar, and is referred to by the acronym MIDASC (for MIDAS Canada).

Table 1 summarizes all the MIDASC experiments from which data are presented in this paper, including the locations of the Millstone Hill transmitter and of the two MIDASC receiving sites. At London, Ontario, the equipment was located in the Physics and Astronomy Building at the University of Western Ontario, and the antenna was a 3 m diameter steerable parabolic antenna mounted on the roof of the building. The other bistatic receiving site was located in Algonquin Provincial Park at the former Algonquin Radio Observatory. Here the receiver was connected to a fully-steerable 46 m diameter parabolic antenna [Jeffery, 1969] using a short cavity mounted helical feed at the prime focus. For all three MIDASC experiments reported on here 1000 μs pulses were transmitted at a prf of 50Hz; the other parameters are shown in Table 1.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Date</th>
<th>Time interval, UT</th>
<th>Transmitter Range to target</th>
<th>Expected delay to target</th>
<th>Aspect angle</th>
<th>Sampling interval</th>
<th>Each record an average over</th>
<th>Records taken every</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9 March 1994</td>
<td>0312–0408</td>
<td>760 km</td>
<td>5070 μs</td>
<td>1.7</td>
<td>20 μs</td>
<td>60 s</td>
<td>2 min</td>
</tr>
<tr>
<td>2</td>
<td>7 April 1995</td>
<td>2109–2140</td>
<td>824 km</td>
<td>5500 μs</td>
<td>3.7°</td>
<td>20 μs</td>
<td>30 s</td>
<td>1 min</td>
</tr>
<tr>
<td>3</td>
<td>8 April 1995</td>
<td>0302–0357</td>
<td>790 km</td>
<td>5265 μs</td>
<td>2.6°</td>
<td>20 μs</td>
<td>30 s</td>
<td>1 min</td>
</tr>
</tbody>
</table>

The transmitter is located at 42.62°N, 71.49°W. The London receiver is located at 43.01°N, 81.27°W. The Algonquin receiver is located at 45.96°N, 78.07°W. The receiver wavelength, λp, is the instability wavelength observed by the bistatic receiver; at Millstone Hill this value is always 34.1 cm.

Figure 1. Magnetic geometry and locations for the MIDASC experiments. Circles centered on Algonquin and Millstone show the horizons at 110 km for the antennas at those sites. Within each circle are shown contours of magnetic aspect angles at 110 km, shaded for all angles less than 1°. Target scattering volumes for each of the three experiments are indicated.

[14] Table 1 summarizes all the MIDASC experiments from which data are presented in this paper, including the locations of the Millstone Hill transmitter and of the two MIDASC receiving sites. At London, Ontario, the equipment was located in the Physics and Astronomy Building at the University of Western Ontario, and the antenna was a 3 m diameter steerable parabolic antenna mounted on the roof of the building. The other bistatic receiving site was located in Algonquin Provincial Park at the former Algonquin Radio Observatory. Here the receiver was connected to a fully-steerable 46 m diameter parabolic antenna [Jeffery, 1969] using a short cavity mounted helical feed at the prime focus. For all three MIDASC experiments reported on here 1000 μs pulses were transmitted at a prf of 50Hz; the other parameters are shown in Table 1.

[15] The map given in Figure 1 shows the three transmitting and receiving sites (Millstone Hill, London, and Algonquin), the locations of the beam-intersection targets for each of the three experiments, and the magnetic aspect angles for Millstone Hill backscatter and for the Millstone Hill - Algonquin bistatic path. Also shown in this figure are the radar horizons for the Millstone Hill and Algonquin antennas. For Millstone Hill the limit of about 4° elevation is required to avoid local interference, while at Algonquin the antenna is constrained to elevation angles greater than 12° due to mechanical steering restrictions. The elevation horizon for London (not shown) is approximately 0°, so virtually all of the region illuminated by Millstone Hill is also accessible to the London receiver.

[16] Experiment 1 was carried out on the evening of 8–9 March 1994 using the 3m dish at London, Ontario. The beam width of this receiving antenna was about 40°, so the scattering region for this experiment was largely defined by the Millstone beam and pulse. The target location was selected to give the smallest possible bistatic aspect angle, in order to maximize the received power. Weak echoes were observed intermittently throughout the early evening, with a brief period of much stronger echoes from 3:50 to 4:10 UT. Experiments 2 and 3 were both carried out with MIDASC at the Algonquin Observatory on the afternoon...
and evening of 7–8 April 1995, during a magnetically active period (KP about 5). Experiment 2 provided a small aspect angle at Algonquin, and a larger value at Millstone Hill. During that experiment, strong echoes were detected at both receivers and attenuation was introduced to minimize saturation effects. Range-power profiles were generally quite sharp at Algonquin with durations close to the pulse length, consistent with the small beam-intersection volume, whereas at Millstone Hill the returns were somewhat more extended, consistent with the low elevation beam passing through a thin (<10 km) scattering layer. Spectra observed at both sites were single peaked.

Experiment 3 was designed to have moderately large aspect angles at both sites (see Table 1). Echoes were obtained at the Algonquin site throughout much of the hour between 3 and 4 UT, whereas only a few sporadic echoes were observed at Millstone. This is the reverse of what might be expected, since the nominal magnetic aspect angle for the Algonquin path was larger than for Millstone. However, it was found that while the Millstone echoes occurred at delays close to those expected for target #3, the Algonquin echoes had delays of between 4600 and 4900 µs, considerably larger than the expected value of 3560 µs. In Figure 1 sections of the 4600 and 4900 µs time delay ellipses have been drawn and labeled as #3a. It can be seen that they lie very close to the zero aspect angle contour for the bistatic circuit. When the combined transmitting and receiving antenna gains are estimated for various locations along these time delay ellipses, it is found that the darkened region, which is close to the line-of-sight direction for the Millstone beam, has a gain at least 15 dB greater than at any other location at those time delays. While there is no way of proving that the echoes come from this #3a target, the available evidence is consistent with that hypothesis, as will be discussed further.

3.2. The 1989 EISCAT Experiment

The tristatic UHF EISCAT radar system operates near 933 MHz. It consists of a transmitter and receiver at Tromsø, Norway, with additional receivers at Kiruna, Sweden and Sodankylä, Finland. Although designed for incoherent scatter observations it has been used for a number of radar auroral studies [e.g., Schlegel and Moorcroft, 1989; Moorcroft and Schlegel, 1990; Jackel et al., 1997]. Technical information about the EISCAT system is available from a number of sources [e.g., Baron, 1986]. A radar auroral backscatter experiment was carried out with the UHF EISCAT system on 14 June 1989. Five E region targets were used in this experiment, with aspect angles ranging from 4.2° to 11.4°. The integration time was 5 seconds at all sites, with care taken to avoid effects due to antenna motion. Further details on this experiment are given by Jackel et al. [1997].

3.3. EISCAT and COSCAT Observations From 1992

The COSCAT radar system [McCrea et al., 1991] uses the EISCAT receivers with an additional 500w 930 MHz CW transmitter located in Oulu, Finland. An array of dipoles produces a beam with a width of 4° in azimuth and 8° in elevation. Although the power and gain are considerably smaller than that of EISCAT, the magnetic geometry is much more favorable, as E region aspect angles as small as 0.5° can be observed from Sodankylä and 1.5 degrees from Kiruna.

In a campaign during 25–28 June 1992 the COSCAT system was used to illuminate a target where Kiruna could observe small aspect angle E region echoes. At the same time, data was collected with EISCAT from the same region using the Tromsø-Sodankylä link. Here we present data only from Kiruna and Sodankylä, avoiding the complications with the Tromsø observations due to its larger scattering volume and lower spectral resolution. This resulted in a database consisting of hundreds of records with 5 second integration times from which spectral information could be obtained. Aspect angles were 1.5° and 4.6° for Kiruna and Sodankylä respectively.

4. Observations

All the observations which will be discussed in this study are summarized in Table 2. For the MIDASC and the 1992 COSCAT/EISCAT data, we have restricted the analysis to data for which the SNR>1. This criterion corresponds quite closely to that used for the 1989 EISCAT data, for which data were retained only if the cross section exceeded 10⁻¹⁵ m² im⁻¹. All the 1992 data presented here were obtained on 25 June 1992. Altogether, the data from all experiments include about 15 different aspect angles, ranging from 0° to 8.4°. All of the spectral information presented in this paper is in terms of the parameters introduced in section 2: moments of the spectra expressed in terms of Doppler velocity (equations 3–6: $v$, $\sigma_v$, $\Sigma_v$, and $S$) and the ACF shape parameters $n_\tau$ and $\tau_\tau$.

As can be seen from Table 2, at small magnetic aspect angles (i.e., for aspect angles less than 3°) the data set includes 55 observations at 440 MHz and 130 at 933 MHz. At large magnetic aspect angles this data set includes almost 800 observations at 933 MHz, but only 26 at 440 MHz, and those from a brief, single experiment. Thus, we can meaningfully contrast the behavior at the two frequencies at small aspect angles but not at large aspect angles. Similarly, we can contrast the behavior at large and small aspect angles at 933 MHz, but not at 440 MHz.

4.1. Parameter Distributions

Figure 2 contains distributions of each of the chosen parameters for 440 MHz (small aspect angle) and for 933 MHz (both small and large aspect angle). The first column of Figure 2 shows distributions of average velocity. There is no reason to expect any particular relationship among these, since the velocity is expected to vary with, for example, flow angle, electric field, and electron temperature. Nevertheless, it is interesting to note that the largest speeds in this study occur at 933 MHz and large aspect angles, with speeds extending right up to 800 m/s. These high speeds at large aspect angles have been observed in previous EISCAT studies [e.g., Schlegel and Moorcroft, 1989; Jackel et al., 1997], although the distributions have not been explicitly presented before.

Standard deviations of the spectra are given in the second column of Figure 2. At 933 MHz, large aspect angle spectra are clearly broader than their small aspect angle counterparts by a factor of almost two. Comparison of small
aspect angle results at 440 and 933 MHz is complicated by the different range of Doppler speeds for each case. As will be shown later, wider spectra tend to be observed at smaller velocities.

[26] In the third column of Figure 2 are the third root of the third moment, expressed in velocity units: $\sqrt[3]{S^*}$. All three distributions have a distinct peak centered around 200 to 300 m/s, and two of the three have a smaller secondary peak at around $-200$ to 300 m/s. The next column of Figure 2 has the corresponding distributions for the quantity $S$, the skewness multiplied by minus the sign of the velocity, as explained previously. These distributions are more scattered than those of $\sqrt[3]{S^*}$, almost certainly the result of the compounded uncertainties resulting from the division of $(\sqrt[3]{S^*})^3$ by $\sigma_v^3$ to get $S$. The predominance of positive values is still apparent, but there is no longer a clear division into two peaks on either side of zero. For these reasons, we will use $\sqrt[3]{S^*}$ and not $S$ in the remaining figures of this paper.

[27] The fifth column of Figure 2 shows the distributions of the ACF shape exponent, $n_v$. Recall that a value of $n_v = 1$ corresponds to a Lorentzian spectrum, while a value of $n_v = 2$ corresponds to a Gaussian. The distributions of all the large and small aspect angle data at 933 MHz have single-peaked distributions centered on a value of $n_v = 1.5$. The distribution at 440 MHz is similar, except for a number of values which exceed 2.0. All of these come from the Algonquin observations of the MIDASC 3 experiment (see Table 2).

[28] In the last column are distributions of the correlation time, $\tau_v$. The large aspect angle 933 MHz data have a very narrowly peaked distribution centered on about 150 $\mu$s. All but one of the 440 MHz observations with correlation times greater than 700 $\mu$s are from the Algonquin MIDASC 3 experiment. If those values are set aside, the remaining distribution is almost identical to that for the 933 MHz small aspect angle observations, with a peak close to 400 $\mu$s. There is no doubt that the MIDASC 3 Algonquin observations all correspond to very long correlation times; the ACFs were observed to fall off much more slowly than for the other 440 MHz data sets. However, since the ACF magnitudes had not decreased very much by the end of the 1000 $\mu$s pulse length, the fitting for values of $n_v$ may have been biased, and that may explain the large number of values of $n_v$ greater than 2.

4.2. Aspect Angle Dependence

[28] In Figure 3 we examine the aspect angle dependence of selected parameters in more detail, plotting parameter averages against aspect angle for each of the 21 different experimental configurations listed in Table 2. In this figure we have used only four of the six parameters from Figure 2. As already explained, we do not use $S$ and since $\tau_v$ and $\sigma_v$ carry essentially the same information, we show only $\tau_v$ in this figure. Open symbols are used for aspect angles less than $2^\circ$, filled symbols for larger aspect angles.

[30] Panel (a) seems to indicate an increase in average speed with increasing aspect angle. This is a somewhat spurious result, entirely due to the 1989 EISCAT observations. Examination of the velocity distributions indicate that this dependence arises from the combination of a spatial variation in the electric field (previous EISCAT studies have shown that the Doppler velocity is proportional to the line-of-sight component of the $F$ region drift velocity [Schlegel and Moorcroft, 1989; Jackel et al., 1997]), and a threshold velocity effect at the higher aspect angles, a consequence of the rapid decrease in echo intensity with increasing aspect angle.

[31] Panel (b) shows a very consistent trend to smaller correlation times (equivalent to larger spectral widths) at larger aspect angles, as also noted in Figure 2. For aspect

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**Figure 2.** Distributions of spectral moments and ACF shape parameters.
angles less than 3° the values of \( \tau_e \) are clustered around 400 ms (with the exception of Algonquin, Experiment #3, which will be discussed separately), while for aspect angles greater than 4° \( \tau_e \) is about 150 ms. Since all but one of the values below 3° aspect angle are for 440 MHz, while all those above 4° are for 933 MHz, this pattern may be partly attributable to the frequency difference. However, the average values found for the 1992 Kiruna data and the Millstone, Experiment 2 data indicate that there is a true aspect angle effect, and that any dependence on frequency is likely less important.

32] The skewness parameter \( S_v^* \) is given in panel (c) of Figure 3. Positive and negative values of \( S_v^* \) were averaged separately, producing the duplicate symbols at the same aspect angle. There is a slight tendency to large values at larger aspect angle, but with considerable scatter. The number of negative values of \( S_v^* \) is surprising. In previous work the skewness of auroral backscatter has, in most cases, had the opposite sign to the mean velocity, corresponding to a positive value of \( S_v^* \) (see equation 6) [Abel and Newell, 1969; Schlegel et al., 1986; Schlegel and Moorcroft, 1989; Schlegel et al., 1990; McCrea et al., 1991; Eglitis et al., 1995; St.-Maurice et al., 1989; del Pozo et al., 1993]. Only Jackel et al. [1997] showed significant numbers of results corresponding to negative values of \( S_v^* \).

33] One of the most interesting results from this study is illustrated in panel (d), which shows the decay exponent, \( n_t \). The average values of this parameter are strongly concentrated around a value of 1.5 (once again with the exception of Algonquin, Experiment #3). This value appears to depend very little on either frequency or aspect angle.

4.3. ACF Shape Parameters \( \tau_e \) and \( n_t \)

34] In this paper we have used an exponential model of the ACF magnitude described in terms of a decay exponent, \( n_t \), and a correlation time, \( \tau_e \) (inversely related to the spectral width, or \( \sigma_v \)). In Figure 4 we explore in more detail the inter-relationship of these three parameters. No significant dependence on frequency or aspect angle was found here, so data from both frequencies and all aspect angles have been combined together in this figure. However, the data from the 5 EISCAT observations sets have been randomly decimated to leave 25 values for each EISCAT data set in this figure. This makes these plots more easily seen, without significantly altering the distribution of the EISCAT data points in the figure.

35] The scatterplot in panel (a) reveals no particularly systematic relationship between \( \tau_e \) and \( n_t \). However, it is evident, as was also seen in Figure 2, that small aspect angles (open symbols) tend to have narrower spectra (larger values of \( \tau_e \)). This panel highlights very clearly the unusual echoes obtained at Algonquin during experiment 3, having very large correlation times and values of \( n_t \) greater than 2.0.

36] As already noted, we expect an inverse relationship between \( \tau_e \) and \( \sigma_v \), and that is found to be approximately true.
To illustrate more precisely the nature of this relationship we introduce a new quantity, \( s \), defined by the relationship

\[
st = \frac{\lambda_p}{\sqrt{2\pi}\tau_p}
\]

where \( \lambda_p \) is the plasma wavelength defined in equation 1. It is easy to show that for a Gaussian spectrum and ACF, \( s \) is equal to \( s_v \). Panel (b) of Figure 4 shows \( s_v / s \) as a function of \( n_t \). If the spectra were all Gaussian the data should lie along the \( s_v / s = 1 \) line. Although there is considerable scatter, on average the data points do approach \( s_v / s = 1 \) as \( n_t \) increases toward 2. The smallest values of \( n_t \) correspond to larger values of \( s_v / s \). Thus, there is an approximate inverse relationship between \( s \) and \( \tau_v \), but the constant of proportionality depends on the value of the decay exponent, \( n_t \).

4.4. Velocity

In Figure 5 we present scatterplots of the ACF parameters and spectral width as functions of velocity. Correlation times are smallest for the smallest speeds (panels (a), (b), and (c)), although the degree of variation is quite different in the different data sets; for the 933 MHz small aspect angle observations (1992, open circles, panel (c)) there are no small speed observations to show such an effect. As expected, the plots of \( s_v \) in panels (d), (e), and (f) show a mirror-image behavior to those of \( \tau_v \), as expected.

What is noticeable in all these panels, but particularly in panels (b) and (e), is that the spread in \( s_v \) is much greater than that in \( \tau_v \). The spectral width is largest at the smallest velocities, as has frequently been reported at both UHF and other frequencies [e.g., Moorcroft and Tsunoda, 1978; Eglitis et al., 1995].

The behavior of \( n_t \) is noteworthy. Evidently spectra are most nearly Gaussian near zero velocity, for both large and small aspect angle observations. At small aspect angle there is evidence of larger \( n_t \) values again near ion acoustic-like velocities.

4.5. Skewness

Figure 6 shows the dependence of the skewness parameter \( S_v^* \) on velocity and spectral width. In the upper row of panels is shown the dependence of \( S_v^* \) on mean velocity. There is no sign of any significant trend in the magnitude of \( S_v^* \) with velocity, and magnitudes are similar for both large and small aspect angles, as already noted earlier. Note the significant minority of negative values, which tend to be more closely concentrated near zero velocity than the positive values. This applies to both small

![Figure 5](image1.png)

**Figure 5.** Scatterplots of (a–c) \( \tau_v \), (d–f) \( s_v \), and (g–i) \( n_t \) versus mean velocity and skewness \( (S_v^*) \) for different experiment data sets, as in Figure 4. Symbols are the same as those used in Figure 3, except that the 1989 Kiruna observations are represented here by open circles.

![Figure 6](image2.png)

**Figure 6.** (a–f) Scatterplots of \( S_v^* \) versus velocity and spectral width \( (s_v) \) for different experiment data sets, as in Figure 4. Symbols used in this figure are the same as Figure 5.
and large aspect angle observations, as was seen indirectly in Figure 2.

[40] The lower row of Figure 6 shows a strong dependence of $S^2$ on $\sigma$, for both large and small aspect angle, a consequence of the normalization of $S^2$ by $\sigma$ (see equation 6). The diagonal dashed lines in these panels are $S^2 = \pm \sigma$, corresponding to values $S = 1$. It is evident that for both frequencies the small aspect angle skewness has a magnitude very close to 1. At large aspect angle the skewness is more variable; the 1989 EISCAT observations average only slightly less than 1, while the 1992 Sodankyla magnitudes are mostly between 0.5 and 1.0.

5. Discussion

[41] Several clear patterns in UHF spectral characteristics have been observed. Placing these results into an appropriate context is, however, not straightforward. There is relatively little theoretical work which includes the kinetic effects likely to be important at UHF wavelengths. The few publications involving kinetic theory [e.g., Robinson and Honary, 1990] generally do not deal with spectral characteristics of the kind we have considered in this study. It is thus necessary to expand the scope of our discussion to consider fluid theory predictions, even though they may not necessarily be directly applicable at UHF.

5.1. Shape of the Autocorrelation Function

[42] The use of an exponential model for the ACF has led to several interesting results. Foremost among these is the discovery that the power spectra at both 440 and 933 MHz are, for the most part, neither Gaussian ($n = 2$) nor Lorentzian ($n = 1$). Instead, the ACF magnitude decay exponent $n$, has an average value of about 2.5, intermediate between the two standard models. In an earlier study of coherent backscatter from the Millstone Hill radar, del Pozo et al. [1993] suggested that a peak in the spectrum of homogeneous stationary turbulence should produce backscatter with a Lorentzian power spectrum (i.e., $n = 1$), and analyzed spectra based on that assumption. However, they did not test the validity of that model for their data.

[43] At HF, Hanuise et al. [1993] and Villain et al. [1996] have fit various models to experimental ACFs and found that while some ACFs were fit by Lorentzians and others by Gaussians, the best fits were often found to be Gaussian at small lags and Lorentzian at large lags, shapes similar to those found in the present study. Villain et al. [1996] have argued that the intermediate ACF shapes which they frequently observed correspond to a situation where the turbulence scale is approximately equal to the plasma wavelength. This interpretation is not easily reconciled with the fact that similarly shaped ACFs are observed at such a wide range of plasma wavelengths, from about 15 m (HF) to 0.34 m (440 MHz) and 0.16 m (933 MHz). It may also be significant that the same ACF shape is observed at both large and small aspect angles, suggesting some sort of universal character to the shape. Work in progress (A. Hamza, D. R. Moorcroft and B. Jackel, 2001) interprets these ACF shape measurements in terms of non-Boltzmann-Gibbs fluctuation statistics.

[44] There is some evidence for spectral shapes to be more nearly Gaussian ($n \approx 2$) at small Doppler speeds (Figure 5). This is most clear at 933 MHz, for both small and large aspect angles. Small aspect angle results also show a tendency to be more Gaussian at speeds larger than $C_e$. Some difference in spectral characteristics at different speeds might be expected, given that the standard two-stream instability only operates above some velocity threshold near $C_e$. Small speed echoes presumably arise from “secondaries” produced by a cascade mechanism [Sudan et al. 1973]. Variations in $n_e$ with Doppler shift may thus be a reflection of the different processes that work at different phase speeds. Other studies [del Pozo et al., 1993; Moorcroft, 1996b] have also observed wider spectra at small phase speeds. However, to our knowledge, our results are the first to show a clear difference in UHF spectral shape (i.e. Gaussian versus Lorentzian) as well.

5.2. Correlation Time

[45] The exponential model used in this paper introduced the correlation time, $\tau_c$, as an alternative to the second moment or other measures of the spectral width. $\tau_c$ is evidently a useful parameter, showing very clear behavior as already discussed. Particularly striking is the constancy of $\tau_c$ over a significant range of large aspect angles, independent of the Doppler velocity.

[46] Eglitis et al. [1995] studied small aspect angle backscatter over a range of radar frequencies from 10.7 m to 16 cm, and found that the spectral width, expressed in frequency units (Hz), was proportional to $k^{-1}$. Equivalently, their results indicate that the Doppler width, in velocity units (m/s), is essentially constant for all irregularity wavelengths. At first sight, their result appears to contradict the behavior found here, that $\tau_c$ is nearly independent of frequency (Figures 2 and 3b) while $\sigma_c$ decreases with increasing frequency (Figure 2). This discrepancy is only apparent. Because of the relatively small number of small aspect angle observations in this study it is necessary to take into account the well-known variation of $\sigma_c$ with velocity. Since most of the 933 MHz small aspect angle observations have large Doppler velocities the frequency comparison should use only the large Doppler velocity observations; when that is done the values of $\sigma_c$ at the two frequencies are found to be virtually identical (5d and 5f), in agreement with Eglitis et al. [1995].

[47] A different perspective can be achieved by expressing spectral widths in hertz rather than the more commonly used velocity units. The choice of velocity units for spectral width corresponds, in a sense, to a physical interpretation of the spectral spread in terms of a range of Doppler shifts within the observation volume. An alternate view is that the correlation time may be the more physically significant quantity, that is, observations of spectral width or correlation time reflect the lifetimes of individual phase-coherent fluctuations in the plasma. In that case the correlation time, $\tau_c$, provides a very direct measure of this property of the plasma instabilities, and is the most appropriate type of measurement. However, the fact that $\sigma_c$ is nearly constant with changes in wavelength (for a given Doppler speed) suggests something different. Eglitis et al. [1995] interpreted this behavior by using results from Hanuise et al. [1993] to argue (in effect) that $\sigma_c$ is independent of wavelength because the fluctuation wavelength is small compared to the plasma correlation length, which in that theory
also implies that the shape of the ACF (and spectrum) is Gaussian. As discussed in section 5.1, this corresponds to $n_r = 2$, a prediction at odds with the average value of $n_r = 1.5$ found in this paper. It may be that the independence of spectral width on wavelength arises from some property of plasma turbulence, but it appears that the appropriate theory must differ in some important aspects from that of Hanuise et al. [1993] and Villain et al. [1996].

5.3. Measures of Spectral Width

[48] The apparently non-Gaussian nature of UHF spectra has some implications when working with standard spectral moments. For values of $n_r$ in the range $1 \leq n_r < 2$, the standard deviation of the corresponding spectrum does not exist; i.e., it is infinite. This is well-known for the particular case of a Lorentzian spectrum ($n_r = 1$). Thus, insofar as this model truly describes the experimental ACFs (and it seems to do that very accurately), the spectral standard deviation is not well-defined. So, although Figure 4b suggests a simple relationship between $\sigma_r$ and $\tau_r$, the actual nature of that relationship is complex, and is probably strongly affected by experimental characteristics, such as receiver bandwidth. [49] Full spectral Width at Half the Maximum value (FWHM) is a frequently used alternative to $\sigma_r$. It tends to be more robust than the standard deviation, and also has the advantage that it exists for all spectra, even those which theoretically have no second moment. For a Gaussian ACF it is easily shown that FWHM $= 2\sqrt{\ln 2}/\tau_r$. For other ACF shapes there will not necessarily be a simple analytical expression relating FWHM and $\tau_r$. [50] It may be desirable (for comparison with other experiments) to have a quantity that gives a measure of the spectral width, and yet is related in some direct way to the correlation time. One possible candidate for that would be $\sigma_r$, as defined in equation 8. For a Gaussian ACF it gives the standard deviation, and for other ACF shapes it provides a value which is a very similar measure of spectral width, yet which has a simple analytical relationship to the correlation time.

5.4. Aspect Angle Dependence of Spectral Width and Correlation Time

[51] Many previous studies of UHF auroral backscatter aspect angle dependence have established the rapid decrease in echo strength with increasing aspect angle [e.g., Moorcroft, 1996a and references therein]. Another study by Foster et al. [1992] observed decreases in Doppler velocity, echo height and aspect sensitivity near an aspect angle of $3^\circ$. The present study shows that the correlation time and spectral width also change abruptly near an aspect angle of $3^\circ$. At aspect angles larger than $3^\circ$ the correlation time is essentially a constant value of approximately 150 m/s. [52] There is still no theory to explain the detailed characteristics of large aspect angle echoes. In general terms, it is likely that mode coupling with unstable waves at small aspect angle provides energy to these large aspect angle waves. Energy is dissipated via damping processes which should be largely independent of the magnetic field, and hence of the aspect angle [Dubois, 1988]. The overall lifetime (correlation time) of an individual plasma wave will be determined by the combined effects of the rate at which energy is given to the wave and the rate at which it is lost. We have just argued that the loss rate should be independent of aspect angle, and we have observed that the lifetime or correlation time is independent of aspect angle. The most likely inference is that the mode-coupled energization of these large aspect angle waves occurs in a time short compared to the loss rate, so that the lifetime is determined by the loss rate, and is thus constant. A less likely possibility is that the energization process is slow, but also independent of the aspect angle.

[53] What is so special about the aspect angle of $3^\circ$? This is an intriguing question to which we can not give a definite answer. However, one possible indication is given by the work of Dubois [1988], who used the standard kinetic theory of Farley-Buneman waves to calculate the growth rate and phase velocity for all aspect angles, including those where the waves are damped. It was found that at UHF the growth rate approached a constant (negative) value, independent of aspect angle, at sufficiently large aspect angles. The transition from a varying to a nearly constant value took place between about $2^\circ$ and $4^\circ$. Although such a theoretical approach is obviously incomplete, it is likely a good indication of the aspect angle range for which the loss rate for such waves becomes essentially independent of magnetic field. [54] A significant difference between large and small aspect angle spectral widths at 933 MHz is suggestive of theoretical work by Hamza [1992] who found that larger aspect angle echoes could be stabilized by frequency broadening, and the required amount of broadening increased with aspect angle. From this perspective, the “break” in spectral character as a function of aspect angle may indicate an upper limit on the amount of frequency broadening which can occur. For example, a broadening factor of $\alpha = \Delta v/v \approx 2$ would stabilize waves for aspect angles only up $3.5^\circ$. This would equivalently correspond to a lower limit on the correlation time $\tau_r$. Our distributions of small aspect angle correlation time (Figure 2) are roughly consistent with this interpretation, showing a “tail” of large correlation times and a relatively sharp cut-off for small correlation times. However, observations of echoes at much larger aspect angles do not seem to be predicted by frequency broadening, nor is there any suggestion as to why these echoes should have significantly different (smaller) correlation times. A more quantitative comparison is not possible, as Hamza [1992] uses a fluid theory to obtain his results, and also notes that the introduction of anomalous collision frequencies can profoundly reduce the amount of broadening required at a given aspect angle.

5.5. Velocity

[55] The small aspect angle 440 MHz velocities do not show the predominance of near ion-acoustic speeds reported in previous studies in that frequency range [e.g., Moorcroft and Tsunoda, 1978; del Pozo et al., 1993]. That is probably because of the relatively small number of observations contributing to this study. On the other hand, there are many 933 MHz observations at large aspect angle, and they show no evidence of any ion-acoustic echoes at all; the velocity distribution (Figure 2) is smooth and broad, extending from $-800$ m/s to $+300$ m/s. This result is consistent with previous large aspect angle studies [e.g., Moorcroft and Schlegel, 1988; Jackel et al., 1997].
Fluid theory results of *Hamza and St.-Maurice* [1993a, 1993b] predict for small aspect angle backscatter a relationship between Doppler velocity and spectral width of the form

\[ v^2 + \Delta v^2 \approx C_s^2 \]  

where \( C_s^2 \) is the ion-acoustic speed. This equation predicts that for a given mean Doppler speed, the spectral width of small aspect angle echoes should be the same at 440 MHz and 933 MHz. Figures 5d and 5f (only the patch of narrower echoes are at small aspect angle) show that this is approximately true.

At a fixed radar frequency equation 9 is also successful at predicting the variation of spectral width as a function of Doppler speed. At small aspect angle the spectral width (1/\( \tau_a \)) is found to be small at large Doppler speeds, and largest near zero Doppler (Figure 5d), as found in previous studies [Hall and Moorcroft, 1988; del Pozo et al., 1993]. At large aspect angle there is little variation with aspect angle (Figure 3b), and no clear dependence on Doppler speed; the 1989 EISCAT observations suggest no variation, while the 1992 Sodankyla observations suggest a maximum near zero Doppler (Figures 5e and 5f). Of course there is no reason to expect large aspect angle observations to follow the small aspect angle prediction of equation 9.

### 5.6. Skewness

Asymmetric spectra are common at UHF, typically with a sharp cut-off at higher speeds and a longer “tail” extending toward zero speeds. This basic shape is observed over the entire range of mean Doppler speeds, and occasionally the low speed tail can extend across zero Doppler shift into velocities with a sign opposite to that of the spectral peak. While it is rewarding to find a spectral feature that is apparently shared by all UHF coherent echoes, the significance of a constant value for skewness is unclear. Some work by Hamza [Private communication] indicates that small aspect angle echoes should have skewness values that for a given mean Doppler speed, the spectral width of small aspect angle echoes should be the same at 440 MHz and 933 MHz. Figures 5d and 5f (only the patch of narrower echoes are at small aspect angle) show that this is approximately true.

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### 5.7. Unusual Observations From the MIDASC3 Experiment

Measurements from the MIDASC 3 Algonquin experiment were unusual in several ways. In the first place, all evidence supports the assumption that these echoes were obtained from the small aspect angle target #3a as shown in Figure 1. That is, the echoes were obtained from a target which was in the sidelobes of both the transmitting and receiving antennas. This observation underscores the problem that even with the use of highly directive antennas, bistatic coherent backscatter can be so strong, and the aspect sensitivity so large, that echoes are detected from small aspect angle almost without regard to the locations of the main beams of the antennas. This effect is more familiar monostatically where it has been used as a design feature in some Millstone Hill experiments [e.g., Foster and Erickson, 2000].

The unusual character of the ACFs for this experiment is highlighted in Figure 4a (open inverted triangles) which shows that, overall, these ACFs had larger correlation times and larger decay exponents than those of any other experiment. In fact, the values of \( n_c \) for this experiment are anomalously large, and are certainly spurious. Theoretically, values above 2.0 correspond to a non-positive power spectrum, which is impossible. They likely result from difficulties in fitting the very slowly decaying ACF magnitudes resulting from the large correlation times. Thus, it is probable that for some reason this experiment had mostly Gaussian ACFs and spectra, in contrast to most of the other measurements in this study. The very long correlation times are surprising. Because these echoes were observed in antenna sidelobes they must have been scattered from waves of large amplitude. One might have expected large amplitude waves to be more strongly driven, to get rid of their energy by mode coupling more quickly, and thus to have shorter lifetimes, leading to broader spectra. These echoes do not fit that picture.

### 6. Summary

The primary result of this paper is the identification of similar spectral parameters at 440 MHz and 933 MHz. Although there is only a slight overlap, the variation of these parameters with aspect angle also appears to be similar at both frequencies. There are significantly larger variations in the Doppler velocity observations, presumably due to differences in the geophysical conditions and the flow angles in the different experiments. These Doppler velocity variations present a striking contrast to the stability of the other parameters. Evidently, the spectral shape (i.e., width/correlation time, skewness, decay exponent) of UHF auroral backscatter is, in some sense, a universal characteristic, not affected by geophysical variations to nearly the same extent as the Doppler velocity.

As in previous studies, the spectral width was found to be greater at small Doppler speeds. Our results show that width increases abruptly with increasing aspect angle to a value about twice as large at aspect angles beyond about 3°. As an alternative to estimates of spectral width, we have found that the correlation time, \( \tau_c \), (which can be thought of as an estimate of the instability lifetime) usefully orders the data. Relationships among the various possible ways of determining spectral width are dependent upon the actual spectral shape (i.e., the value of \( n_c \)), and it appears that the appropriate approach to the measurement of spectral width/lifetimes is dependent on the interpretation to be put on the measurement.

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